Retuning of PID Controllers using Closed-Loop Data for Self-Regulating and Integrating Processes

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Abstract

In this paper, a PID (Proportional, Integral, Derivative) controller retuning method is designed to increase its existing installed performance appropriate for a wide range of self-regulating, non-minimum phase and integrating processes with dead time. Improved closed-loop responses can be expected for processes with first- and second-order dynamics. The method developed uses routine closed-loop operating feedback data from an existing PID controller and estimates a surrogate ARX (Auto-Regressive eXogenous) parametric model with a pre-defined structure and process delay. By replaying back both the white noise load disturbance (i.e., the regression residuals) and any setpoint perturbations, the closed-loop behavior of the PID controller is simulated in a brute-force grid or sample-based search with the three PID tuning parameters systematically incremented. The best PID settings are selected from the simulated cases or scenarios which minimize the output-error variance (setpoint minus process variable) subject to input-move-error variance limits or bounds which constrain or suppress the movement of the manipulated process input or controller output. The surrogate ARX model can be easily updated from any new data set regularly, or on demand, triggered by unexpected model

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prediction performance and the PID programmatically retuned. Actual laboratory experiments are given to show the effectiveness and simplicity of the PID retuning in handling processes of different characteristics i.e., flow, temperature, and pressure as well as simulated level or holdup control.

Keywords: closed-loop estimation, close-loop data, sample-based search, single-loop PID control, digital twin, cyber-physical, smart tuning.

1. Introduction

PID controllers are the most extensively used regulatory control algorithms in industrial processes, particularly within the chemical process industries. PID controllers have been foundational in industrial control for over seventy years (O'Dwyer, 2009; D. Chen and Seborg, 2002)[1, 2]. They are implemented across various control systems, including distributed control systems (DCSs), programmable logic controllers (PLCs), and supervisory control and data acquisition systems (SCADAs). Their widespread use is attributed to their simple structure, robustness, broad applicability, and ease of online retuning (Zhu et al., 2022; Joseph et al., 2022; Somefun et al., 2021)[3-5]. Over 95% of industries utilize PI/PID controllers (Ghoisiya et al., 2018)[6], with earlier studies indicating that 80% of industrial controllers are P-only/PI, while 20% employ full PID control (Luyben and Luyben, 1997)[7].

The performance of PID controllers depends heavily on the proper tuning of three key parameters: proportional, integral, and derivative. Initially tuned with heuristic methods like Ziegler-Nichols[8], these controllers have evolved to address industrial challenges such as non-linearities and time delays Shamsuzzoha and Skogestad (2020)[2]. Despite the availability of various tuning rules, selecting the best method is still challenging, especially for complex processes with non-minimum phase behavior and dead time. Advances in computational power have made optimization-based approaches, including gradient-based and derivative-free techniques, more viable Issa et al. (2019)[9]. Research has focused on non-gradient algorithms like particle swarm, as well as meta-heuristics like genetic algorithms. While data-driven monitoring and automated retuning methods have emerged (Munaro et al., 2023)[10], reliable tuning formulas are still limited, leading to poorly tuned controllers.

Modern approaches utilize model-driven optimization and adaptive techniques to fine-tune PID control parameters in real-time, significantly enhancing system robustness (Joseph et al., 2022; Issa et al., 2019)[4, 9]. Looking ahead, the integration of machine learning (ML) and artificial intelligence (AI) into PID tuning presents immense potential. These technologies promise to further improve adaptability and system performance by enabling real-time monitoring and automated tuning, effectively meeting the increasing industrial demands for greater efficiency and reliability (Coutinho et al., 2023)[11].

This paper introduces a novel retuning methodology to enhance PID controller performance across various industrial processes. It utilizes a brute-force sample-based search method to systematically explore the parameter space and simulate closed-loop responses, closely mimicking real-world operations through a digital twin. This derivative-free approach avoids local minima and improves tuning accuracy and reliability by optimizing PID settings using routine closed-loop feedback data. The use of a digital twin further enhances the method's effectiveness in replicating actual process behavior.

2. Literature Review

For mor than seventy years, proportional-integral (PI) and proportional-integral-derivative (PID) controllers have been crucial in control engineering (O'Dwyer, 2009; Chen and Seborg, 2002)[1]. Surprisingly, only in the past two decades has significant academic interest surged around PID controllers (Coutinho et al., 2023; Åström and Hägglund, 2006)[11, 12]. Joseph et al., (2022)[4] provided a comprehensive overview of classical and modern PID tuning strategies covering methodologies from conventional techniques to innovative approaches. The article emphasizes their practical applications across various domains, highlighting the evolution and significance of PID tuning methods in improving control system performance. Originally tuned using heuristic methods like Ziegler-Nichols and Cohen-Coon, PID controllers have faced industrial challenges, such as non-linearities and time delays, necessitating advancements Joseph et al., (2022)[4].

Modern approaches arose, utilizing models for precise tuning and optimization algorithms for automated parameter adjustment (Issa et al., 2019)[9]. Real-time adaptive techniques enhanced control by responding to dynamic process behaviors, while iterative methods employed experimental data to meet exact control objectives. Across various sectors, such as chemicals and automotive industries, case studies highlighted customized tuning strategies that tackled specific industrial challenges, illustrating the transition from heuristic methods to advanced, model-driven approaches. This section highlights some of the recent publications in the area of PID tuning research.

(O'Dwyer, 2009)[1] in his Handbook of PI and PID Controller Tuning Rules made a thorough collection of tuning guidelines for proportional-Integral (PI) and Proportional-Integral-

Derivative (PID) controllers, which are commonly utilized in industrial process control. Covering over seventy years of research from 1935 to 2005, the book categorizes and elaborates on various tuning methods. It introduces a standardized notation for these rules, facilitating their application across diverse systems. Important topics include controller's architecture, process modeling, and the performance and robustness of control loops. (Bombois et al., 2005)[13] also provided practical guidance on selecting the optimal method based on model requirement and disturbance characteristics comparing open-loop and closed-loop methods for identifying Box-Jenkins models, focusing on estimation variance. By introducing a novel variance analysis framework, the authors show that while closed-loop identification is generally preferred for its stability, open-loop approaches can offer comparable or superior variance under certain conditions.

Another review article published by (Borase et al., 2021)[14] that provides a comprehensive survey of PID controllers, emphasizing their central role in industrial process control due to its simplicity and reliability. The review evaluates classical and modern PID tuning methods, highlighting their strength and limitations. Classical techniques, such as Ziegler-Nichols (Z-N), Cohen-Coon (C-C), are widely used for their intuitive and practical approaches, with Z-N offering straightforward guidelines and C-C providing improved stability margins for processes with time delays. However, these methods often face challenges in systems with nonlinearity, long dead times, or time-varying dynamics. To address these issues, optimization-based strategies have been developed, utilizing techniques like genetic algorithms, particle swarm optimization, and gradient-based methods. These modern approaches optimize controller's parameters by minimizing cost functions that balance performance metrics such as rising time, overshoot, and robustness. While they offer enhanced adaptability and performance in complex systems, they are constrained by computational requirements and the need for accurate models. The review underscores how these advancements in PID tuning continue to meet the diverse demands of modern control systems, solidifying PID’s relevance across a broad range of applications, traditional industries to advanced robotics and renewable energy systems.

Xinqing Gao et al. (2017)[15] proposed a data-driven method that integrates performance assessment and retuning of PID controllers, eliminating the need for separate steps. By relying solely on process data, the approach avoids detailed modeling requirements and is robust to noise and disturbances. It introduces a performance benchmarking metric and an automatic retuning mechanism to optimize controller performance. The method's practicality is validated through simulations and real-world case studies, demonstrating its effectiveness for industrial PID retuning applications

In their paper, J. Park et al. (2020)[16] present an Arduino microcontroller-based temperature control lab (TCLab) as a benchmark for modeling and control methods, emphasizing its practical applications in real-world scenarios, including cycle time and discrete sampling intervals. They explore four modeling approaches: a physics-based lumped parameter model, a first-order plus dead-time (FOPDT) model, an autoregressive exogenous input (ARX) model, and a Hammerstein model with an artificial neural network (ANN). The paper also showcases an optimization technique for PID controller tuning, yielding a 5.4% performance improvement. Additionally, the study compares Model Predictive Control (MPC) using different models, highlighting the relative strengths and weaknesses. The TCLab’s widespread use in educational and industrial settings, with over 3000 units distributed, underscores its value as a practical learning tool for process control.

Coutinho et al., (2023)[11] published a novel methodology for automatic tuning of multi-loop PID controllers using Bayesian Optimization (BO). It highlights BO as an efficient, data-driven global optimization method, particularly effective for PID controllers in Multi-Input Multi-Output (MIMO) processes. A systematic approach defines the optimization domain by integrating sequential loop closing, system identification, and model-based tuning relations. Each loop is tuned sequentially, starting with the fastest, to ensure closed-loop stability and account for process interactions. Performance is evaluated using Integrated Absolute Error (IAE) and Total Variation (TV) of the controller output, balancing tracking performance and control effort. The methodology, tested on a non-linear evaporator process, demonstrates significant performance improvements over traditional tuning methods. Future research suggestions include enhancing BO scalability, reducing online experimental costs, and incorporating robustness measures. This approach enhances the efficiency and safety of PID controller tuning without requiring extensive prior knowledge or detailed MIMO process models.

Munaro et al., (2023)[10] presented a data-driven methodology for performance assessment and retuning of PID controllers. It uses set-point response data to compute the Integral of Absolute Error (IAE) and applies statistical tests to monitor performance. If performance degrades, new controller gains are estimated using closed-loop data and a reference model. Tested on pilot plants with flow, level, and pressure loops, this methodology handles noisy environments and various set-point changes effectively. Performance monitoring is done via normalized IAE values using an exponentially weighted moving average (EWMA) control chart, with performance degradation triggering retuning. New parameters are estimated using the Optimal Controller Identification (OCI) method, validated through statistical tests. The approach is robust, entirely data-driven, and does not require a process model, making it applicable to a wide range of industrial control loops. The methodology is validated through pilot plant applications, demonstrating its effectiveness in recovering performance and handling practical aspects.

3. Retuning Methods

Rule-based PID tuning methods can be divided into two categories: (1) process model from open or closed loop data and (2) closed-loop frequency and gain response analysis. Most of the open-loop tuning rules and many of the closed-loop tuning rules derive the PID tuning parameters by referring to a process model that is first order or second order plus dead time model (FOPDT or SOPDT). In this case, the performance of the PID tuning is highly dependent on the process identification result. However, breaking the control loop might not be welcomed in a real operation and the process identification from the closed-loop data may not be as accurate as the open-loop identification.

There are methods in the second category that refer to the frequency response of the closed-loop data instead of constructing a closed-form process model. Zeigler-Nichols closed-loop tuning requires sustained oscillation data to obtain an ultimate gain (K_u) and ultimate period (P_u) S. Skogestad (2023)[17]. To avoid driving a process to the limitation of the stability region to obtain the sustained oscillation data, a relay method is introduced by A. Patel et al. [18] . Shamsuzzoha and Skogestad (2020)[2] proposed the SIMC tuning method using closed-loop overshoot response data.

4. Methodology

A novel and straightforward brute-force sample-based search methodology is described to retune a PID controller using closed-loop feedback operating data for both self-regulating and integrating types of processes. The methodology is general and formulated for single-input and single-output (SISO) feedback control systems. The retuning is performed in an offline environment using closed-loop simulation with the ARX model and technically does not require any special interventions or testing to be performed on the physical system or plant. The brute-force sample-based search simply steps through the PID settings (Kp, Ti, Td) respectively beginning from a lower bound and ending at its existing or default PID setting and then starting from its existing PID setting and ending at its upper bound where the total number of steps for each PID setting calculated by the formula below specific to Kp' as an example:

$$INT\left(\frac{Kp' - L}{SL}\right) + 1 + INT\left(\frac{U - Kp'}{SU}\right) + 1$$

This formula represents the total number of steps required to iterate, traverse or loop from a user-specified lower (L) bound via its default PID setting to its upper (U) bound with a lower and upper related step-size (SL and SU) and then adding one (1) to account for the starting values.

This paper discusses the closed loop PID retuning by demonstrating its capability to handle various processes characteristics i.e., flow, temperature, pressure and level. The methodology consists of two main components: closed-loop process identification and offline brute-force search for PID tuning parameters. The summarized step by step procedure and corresponding block diagram are presented below and the detailed descriptions are shown in following sections:

1. A SPO data file with the data-vector names of “sp”, “pv” and “op” in any order including both training and testing data sets aligned one after the other. A PID file containing the default PID settings, their lower and upper bounds and step sizes as well as the PID equation type or form, sampling period duration and lower and upper bounds for the process input and output respectively.
2. A SISO ARX model is identified and estimated for its dynamic model structure (m, n, k) and parameters (a1..am and b0..bn).
3. Identification and estimation are performed to find the best candidate SISO ARX model given its output denominator degree (m), input numerator degree (n) and dead-time or time-delay (k).
4. Simulate the PID using the pseudo-white noise load disturbance time series representing the SISO ARX prediction errors or residuals and the known setpoint, target or reference disturbance signals. The simulated or predicted input and output time -series should adequately match the actual input (op, t) and output (pv, t) time series.
5. Retune the PID controller via a brute-force grid search to find the best tuning by minimizing the 1-, 2- or oo-norm of the output errors (sp,t – pv,t) subject to an upper limit on its 1-, 2- or oo-norm of the input-move-errors (op, top,t-1) for self-regulating processes. For integrating processes, we minimize the 1-, 2- or oo-norm of (op,t – op,t-1) subject to an upper limit on the 1-, 2- or oo-norm of (sp,t – pv,t).

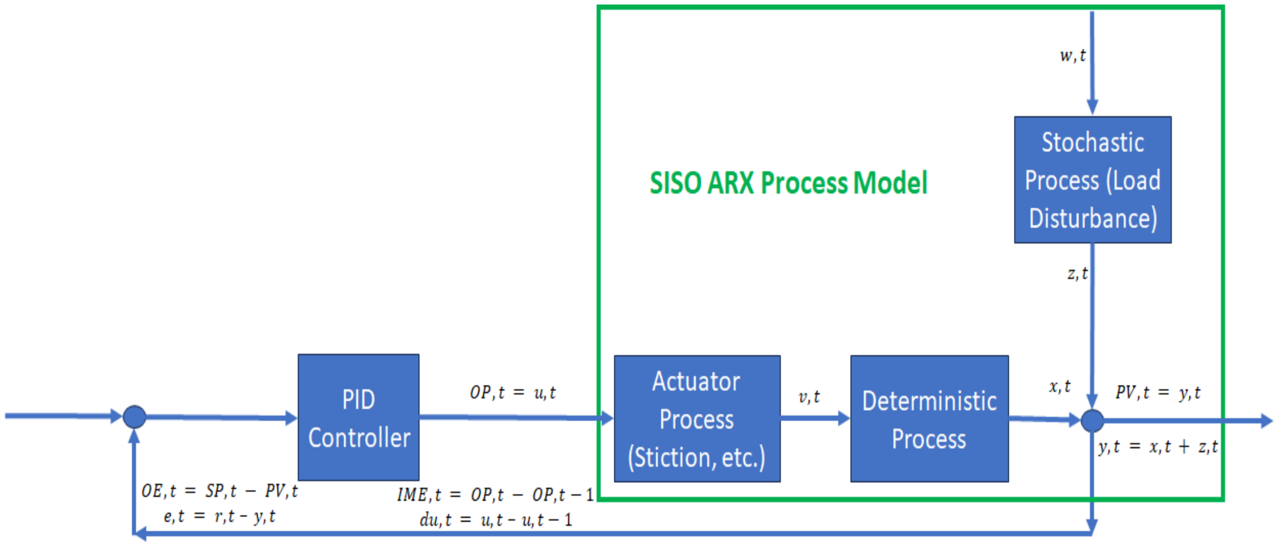

Figure 1. Block Diagram of PID-Controlled SISO ARX Process Model with Disturbances.

And interestingly, even though the single-input and single output linear dynamic process model is most likely structurally over-parameterized, the phenomenon of self-regularization helps to inherently prevent over-fitting Du et. al. (2022) [19]. That is, redundant parameters tend to converge to zero (0.0) as more data becomes available and the remaining parameters tend to converge to their true low-order system parameter or coefficient values without the requirement for explicit parameter regularization via 1- (absolute, Manhattan) or 2- (squared, Euclidean) norm penalty-errors or -elastic (artificial) variables.

4.1 PID Equations

There are many derivations of the PID formula rooted in the original continuous equation detailed in Aström and Hägglund (1995)[20] . For implementing PID controllers in modern digital control platforms such as DCSs (Distributed Control Systems) or PLC (Programmable Logic Controllers), two popular discrete forms are widely used in industry. One is the positional form (Equation 1) and the other is the velocity form (Equation 2), which are interchangeable if the controller output or process input do not exceed their lower and upper bounds.

$$OP_t = OP_{bias} + K_p[OE_t + \frac{\Delta t}{\tau_I}\sum_1^t OE_t + \tau_D \frac{OE_{t-1} - OE_t}{\Delta t}] \quad (1)$$

$$OP_t = OP_{t-1} + K_p[(OE_t - OE_{t-1}) + \frac{\Delta t}{\tau_I} OE_t + \frac{\tau_D}{\Delta t}(OE_t - 2OE_{t-1} + OE_{t-2})] \quad (2)$$

where, $OE_t = SP_t - PV_t$ and is the output error.

Whereas the positional form calculates the controller output position (OP), the velocity form calculates the change in controller output (ΔOP). Although the positional form is more straightforward to understand as its P, I, and D terms are directly translated from its original continuous form, the velocity form has several advantages from the convenience perspective such as no additional logic is required for anti-reset windup as reported in Ljung (1999)[21].

In addition to the choice of the positional and the velocity form, there is another popular derivation regarding the derivative kick that appears on the derivative terms in the PID equation where it is also possible to replace OE in the derivative term with the PV. This modification can reduce the chance of abrupt increments in the controller output when setpoint changes are made. However, in this study, the velocity form without the derivative kick-free form is used i.e., Equation 2 only.

4.2 Closed-Loop Identification and Estimation

The success of various PID tuning algorithms strongly relies on the accuracy of the process model. Notably, the IMC-based tuning rules directly calculate the P, I, and D tuning parameters using the equations as a function of the process model (K, τ and θ). Additionally, this tuning rule requires an approximation of the process model to be of the first or second order plus dead time (FOPDT or SOPDT) form whereas this proposed method is agnostic to both the process and controller model forms. Furthermore, the accuracy of the process model can always be worse in closed-loop identification than in open-loop identification. Thus, these tuning rules might be overly sensitive to the result of process model identification. The proposed method minimizes the chances of losing process information by using any type of higher-order process model directly without requiring any additional approximation to the first-order process model. In addition, this newly proposed method uses closed-loop routine operating data (with feedback) and may fit ARX process models (Eqn 3) although Box-Jenkins models may also be used. Both ARX and Box-Jenkins models have proven consistency in closed-loop identification Flehmig et at. (2008); Voda et al. (1994); Jahanshahi and Skogestad (2013)[22-24]. The ARX form is:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_m y_{t-m} + b_1 u_{t-1-k} + b_2 u_{t-2-k} + \cdots + b_n u_{t-n-k} \quad (3)$$

where m and n represent the number of ARX coefficients multiplying the same number of past output (y) and input (u) measurements respectively. The parameter k denotes the process time delay or dead-time, expressed in sampling intervals, and represents the delay between an input change and its observed effect on the output – excluding the inherent sampling delay.

4.3 Replaying Setpoint and Load Disturbance

The proposed method uses the digital twin of the actual process to iteratively search for the optimal PID tuning. The ARX model identified from the closed-loop data is used as the base model of the digital twin. This identification may be distributed across multiple process models with appropriate chances of occurrence for a robust controller performance design, i.e., including a family of process models. The next step is to evaluate the potential PID tunings through the digital twin by re-playing the same past setpoint and load disturbance (Equation 4) as in the process data with:

$$z_t = y_t - x_t \qquad (4)$$

where, x_t represents the ARX model output for time-step t. This replay of the closed-loop data also describes the actual process more accurately.

4.5 Brute-Force Sample-Based Search Method

To find the optimal PID parameters, a straightforward brute-force sample-based searching method is used. The search method simply scans all the possible PID tuning performances by evaluating the range of PID tuning combinations with the process digital twin. The upper and lower bounds of the P, I, and D tuning are set by the user. It can be set based on the current PID tuning as a retuning purpose or it can be tested for an unexplored range for an experiment. The computational time of the exhaustive searching method can be longer than the gradient-based optimization although it is easily parallelizable as each PID parameter set increment is independent from the other PID increments. However, the computational time is not a constraint of the PID controller tuning tasks as opposed to the real-time estimation problem that needs to return the solution faster than the pace of real-time. On the contrary, one of the major advantages of the exhaustive searching method is that it always guarantees a global minimum given the pre-defined search space, unlike the other optimization methods. By considering the goal of PID tuning and the small size of this optimization problem, an exhaustive search has minimal computational time. If computational time were a concern, the exhaustive search could also be used as a multi-start method in conjunction with a gradient-based optimizer to quickly find local solutions.

Two different types of objective functions may be considered for PID tuning (Equations 5, 6). The objective functions are variations of the PID control performance indices known as ISE (Integral Squared Error) and IAE (Integral Absolute Error). The objective function consists of the output-error (OE) term which appears in ISE and IAE, and the input movement (IM) term. The optimization solution of ISE combined with input movement (or, rate of change) has been analytically derived and investigated in Tchamna and Lee (2018)[25] and is the simplest form of move suppression. These multi-objective functions can be expressed in two different ways. One is Archimedean and the other is the lexicographic form (or goal programming), which is shown in Equations 5, 6 and Equations 7, 8, respectively.

$$L_2 - norm: \quad \min_{Kc,\tau_I,\tau_D} J = \sum w_{OE}\{SP(t) - x(t)\}^2 + w_{IM}\{OP(t) - OP(t-1)\}^2 \tag{5}$$

$$L_1 - norm: \quad \min_{Kc,\tau_I,\tau_D} J = \sum [w_{OE}|SP(t) - x(t)| + w_{IM}|OP(t) - OP(t-1)|] \tag{6}$$

where w is the weighting factor for each term in the objective function denoted in the subscription (OE and IM).

$$L_2 - norm: \quad \min_{Kp,\tau_I,\tau_D} J = \sum\{SP(t) - x(t)\}^2 \tag{7}$$

$$\text{Subject to, } \sum\{OP(t) - OP(t-1)\}^2 < UB_{IM}$$

$$L_1 - norm: \quad \min_{Kp,\tau_I,\tau_D} J = \sum |SP(t) - x(t)| \tag{8}$$

$$\text{Subject to, } \sum |OP(t) - OP(t-1)| < UB_{IM}$$

where UB is the upper bound of the input movement (IM) which may be initially set by the centroid PID performance.

Either the Archimedean or lexicographic form of the objective function can be used for PID controller tuning. In terms of convenience, the lexicographic form is easier to use because it requires one user input parameter, UB_{IM}, as opposed to the Archimedean form that requires two weighting factors on both OE and IM terms.

5. Results

In this section, we examine two distinct pilot plants that include flow, pressure, and temperature loops operating in semi stable condition. The proposed methodology is assessed for each scenario, considering several practical considerations. To provide a concise comparison across all case studies, Table 1 summarizes the default and retuned PID controller performance metrics. The table reports the standardized output-error and input-move-error norms before and after retuning, along with the relative percentage improvement in output-error. This overview highlights the consistent improvements achieved by the proposed retuning method across self-regulating and integrating processes.

Table 1. Summary of default and retuned PID performance across self-regulating case studies.

Case Study	Actual Default OEnorm	Actual Default IMEnorm	Actual Best OEnorm	Actual Best IMEnorm	Improvement OEnorm (%)
PCT40 Flow PID	0.836162	0.010304	0.635537	0.004712	28% ↓
PCT40 Pressure PID	1.139950	0.034072	0.728248	0.062497	26% ↓
PCT40 Temperature PID	0.168450	0.023042	0.123598	0.053948	30% ↓
TCLab Temperature PID	1.826942	3.891286	1.521849	1.445109	16.7 ↓

5.1 Experimental Case Studies

In the field of process control education there several popular experimental devices and we have chosen Armfield Limited's multi-function process control teaching system called PCT40 described in Armfield (2005)[26] and the Temperature Control Laboratory (TCLab) found in

Oliveira and Hedengren (2019)[27] from Brigham Young University which are well-known resources that facilitate hands-on learning. Both systems provide practical tools for students as well as engineers to engage with and understand basic and advanced process control techniques. The PCT40 is used for self-regulating flow, pressure and temperature PID control and the TCLab is used for self-regulating temperature PID control only. The integrating level PID control case study is a virtual simulation of pump-drained surge vessel as no physical system was available.

5.2 Flow PID Control with PCT40

The Armfield PCT40 flow control process measures water flow in millilitres per minute (ml/min) using a proportional turbine sensor and adjusts or actuates the speed of a peristaltic pump expressed as a percentage (%) where the supply of water is from the city municipality. This experiment features a sampling interval, cycle time or time-period duration of 0.5 seconds, a default proportional band of 75, and a default integral time (Ti) of 15 seconds. Notably, no derivative action was applied, which is common practice for flow PID controllers as they have fast dynamics. The default proportional band translates into a default controller gain of 0.044444 via the following formula incorporating the appropriate actuator and sensor scalings i.e., Kp = 100 / (75 / 100) / (1500 - (-1500)) = 0.044444.

In figure 2a we plot the actual default PID controller's closed-loop system behavior with twelve (12) setpoint step-changes trending the setpoint (SP), process variable (PV) and controller output (OP) responses for a total of 4,383 time-periods with a 0.5 second time-period duration as mentioned. The standardized L2-norm of the output-error (SP,t – PV,t) is 0.836162 and 0.010304 for the standardized 2-norm input-move-error (OP,t – OP,t-1). Figure 2b is the simulated version of this plot with the default PID tuning parameters and simulating after fitting an over-parameterized SISO ARX dynamic model with a five (m = 5) degree or order output lag, four (n = 4) degree input lag and a one (k = 1) degree of dead-time corresponding to a single 0.5 second time-delay which aligns with the input-output model that has the smallest or minimum sum-of-squares of errors (SSE) i.e., sum of (y,t – yp,t)^2 where y,t = PV,t and yp,t is the model prediction. The simulated standardized L2-norm output-error and input-move-error are 0.845907 and 0.010274 respectively which match very closely to the L2-norm errors found in figure 2a validating the SISO ARX model.

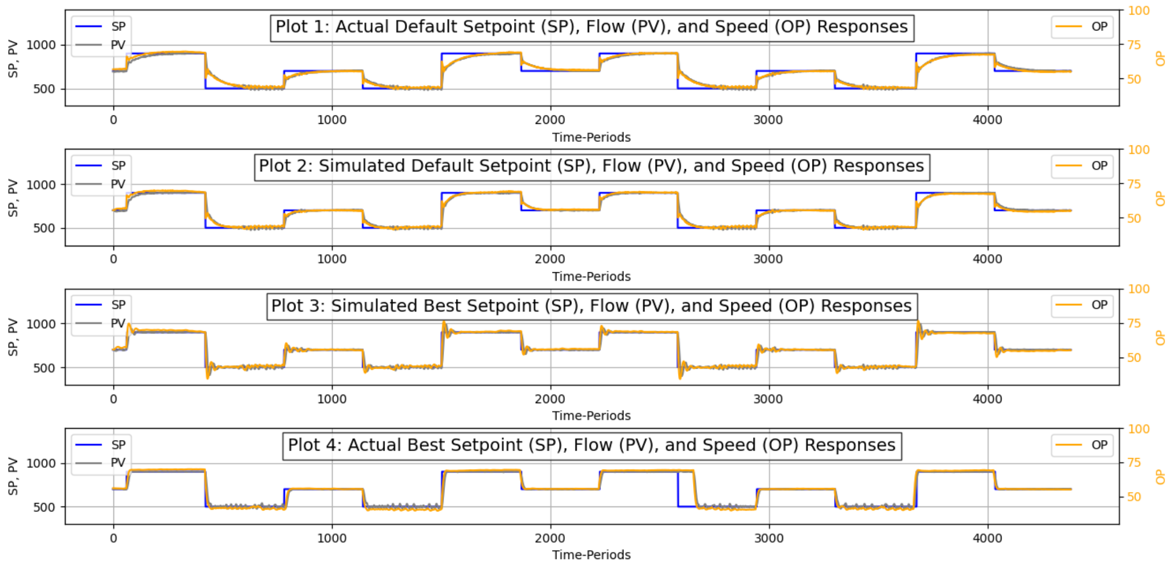

Figure 2. PCT40 Flow PID Retuning Plots.

Figure 2c trends the simulation of the closed-loop system using the SISO ARX model identified and estimated from the data in figure 2a with the best PID controller settings or parameters determined by our PID retuning methodology which are Kp = 0.01 = 100 / (333.333333 / 100) / (1500.0 - (-1500.0)) and Ti = 0.8 seconds. The standardized 2-norm input-move-error upper limit for the retuning is chosen to be one-half of the actual standardized 2-norm IME of approximately 0.005 i.e., ~ 0.010304 / 2. The standardized 2-norm of the output-error (SP,t – PV,t) is 0.596545 and 0.004725 for the standardized 2-norm input-move-error (OP,t – OP,t-1) which shows a close agreement with the chosen input-move-error upper limit and has decreased the output-error by almost 29% i.e., (0.836162 - 0.596545) / 0.836162 * 100 even though we have reduced the input-move-error variation by over 100%. Figure 2d validates these results when the PCT40 closed-loop experiment is repeated less than two-hours after the first closed-loop experiment is performed for figure 2a. The standardized 2-norm output- and input-move-error are 0.635537 and 0.004712 respectively. These 2-norm results are similar to those anticipated from figure 2c except that the reduction in the output-error variation seems to be somewhat over-predicted which can be expected as the time of day is different and the demand for city water can fluctuates causing upstream pressure swings, disturbances, uncertainty, etc.

5.3 Pressure PID Control with PCT40

The Armfield PCT40 pressure control process measures the internal pressure of a sealed-vessel in mmHg and utilizes a variable speed positive displacement gear pump (%) to drive the pressurized flow of water upstream of the supplied water. The water discharge passes through a smaller diameter hole or orifice before eventually draining at atmospheric pressure into the sewer line. A piezoelectric pressure sensor located upstream of the orifice and the pump is controlled by a default PID controller with the settings of a 5-second cycle time or execution interval, a proportional band (PB) of 200.0, and an integral or reset time (Ti) of 15 seconds. No derivative action is necessary as this is a relatively fast process loop and is similar to flow control loops. The default proportional band translates into a default proportional gain of 0.0625 via the following equation: Kp = 100 / (200 / 100) / (400 - (-400)).

In Figure 3a, we illustrate the behavior of the default PID controller's closed-loop system. This includes twelve (12) setpoint step-changes, showing the trends for the setpoint (SP), process variable (PV), and controller output (OP) responses over a total of 723 time periods, each lasting 5.0 seconds. The standardized 2-norm of the output error between the setpoint and process variable (SP,t – PV,t) is 1.112204, while the standardized 2-norm of the output error between consecutive controller outputs (OP,t – OP,t-1) is 0.038826. Figure 3b presents the simulated version of the system using the default PID tuning parameters. This simulation was conducted after fitting an over-parameterized SISO ARX dynamic model, which includes a five-degree (m = 5) output lag, a four-degree (n = 4) input lag, and a one-degree (k = 1) dead-time corresponding to a 5.0 second time delay. This model aligns with the input-output model that has the smallest sum-of-squares of errors (SSE) calculated as the sum of (y,t – yp,t)^2 identical to the flow control example. The simulated standardized 2-norm output error and input move error are 1.139950 and 0.034072, respectively, closely matching the 2-norm errors found in Figure 3a thereby validating the SISO ARX model.

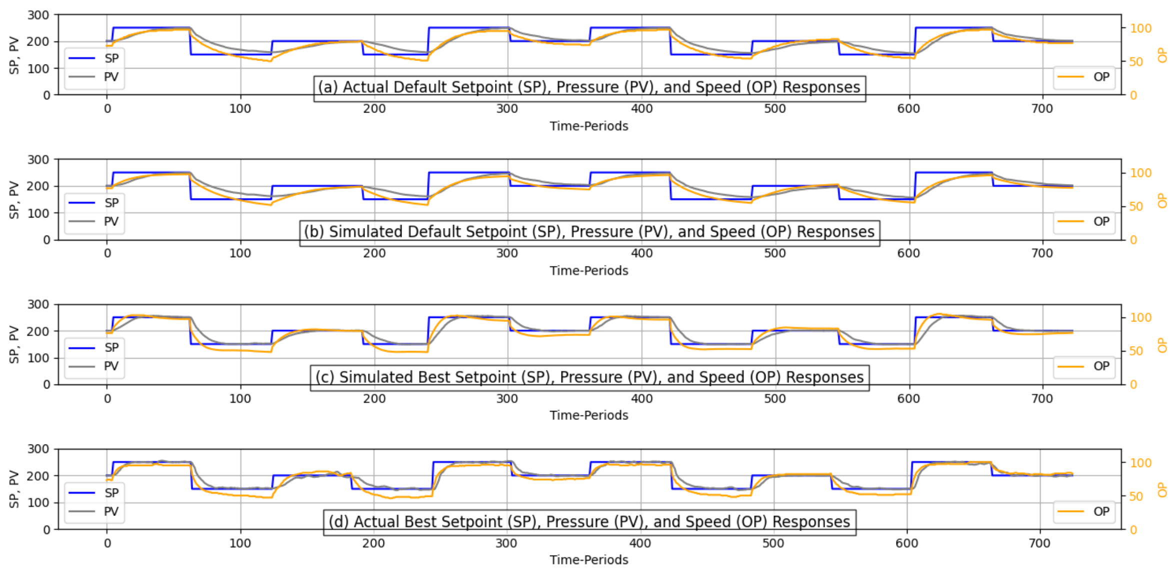

Figure 3. PCT40 Pressure PID Retuning Plots.

Figure 3c shows the simulation of the closed-loop system using the SISO ARX model, which was identified and estimated from the data in Figure 3a. The best PID controller settings were determined by our PID retuning methodology where Kp = 0.1125 = 100 / (111.1 / 100) / (400 - (-400)) and Ti = 10.1 seconds. For the retuning, the standardized 2-norm input-move-error (IME) upper limit was set to approximately double the actual standardized 2-norm IME of 0.07 (i.e.,

~ 2 * 0.038826). The standardized 2-norm of the output-error (SP,t - PV,t) is 0.818852 and 0.0656251 for the standardized 2-norm input-move-error (OP,t - OP,t-1) which shows a close agreement with the chosen input-move-error upper limit and has decreased the output-error by at least 26% i.e., (1.112205 - 0.818852) / 1.112205 * 100 which is expected as we have increased the input-move-error variation by circa 100%. Figure 3d validates these results when the PCT40 closed-loop experiment is repeated less than two-hours after the first closed-loop experiment is performed for figure 3a. The standardized 2-norm output- and input-move-error are 0.728248 and 0.062497 respectively. These 2-norm results are similar and consistent to those anticipated from figure 3c except that the reduction in the output-error variation seems to be somewhat under-predicted which can be expected as the time of day is different and the demand for city water can fluctuates causing upstream pressure swings, disturbances, uncertainty, etc.

5.4 Temperature PID Control with PCT40

The Armfield PCT40 apparatus for our temperature control loop heats water in a vessel where water enters from the bottom and exits as an overflow at the top as an overflow. The inflow of water is regulated by a pressure regulator and a proportional solenoid valve (PSV) set at 50%. A type K temperature sensor is positioned near the top-middle of the vessel and a 2 KW heating element actuated by a solid-state relay (SSR). The heating element operates with time-proportioned or pulse width modulation (PWM) on a 10 second cycle time via the SSR.

A sampling interval of 10 seconds, a proportional band (PB) of 75, and an integral time (Ti) of 30 seconds were used in the first experiment, with no derivative action due to minimal process dead time. The experiment began with a setpoint of 40°C, and the setpoint was nominally perturbed every 15 minutes varying between 30°C and 50°C. The default proportional band corresponds to a default gain of 0.333333, which is calculated using the formula Kp = 100 / (75 / 100) / (200 - (-200)) considering the appropriate scaling for both the actuator and sensor.

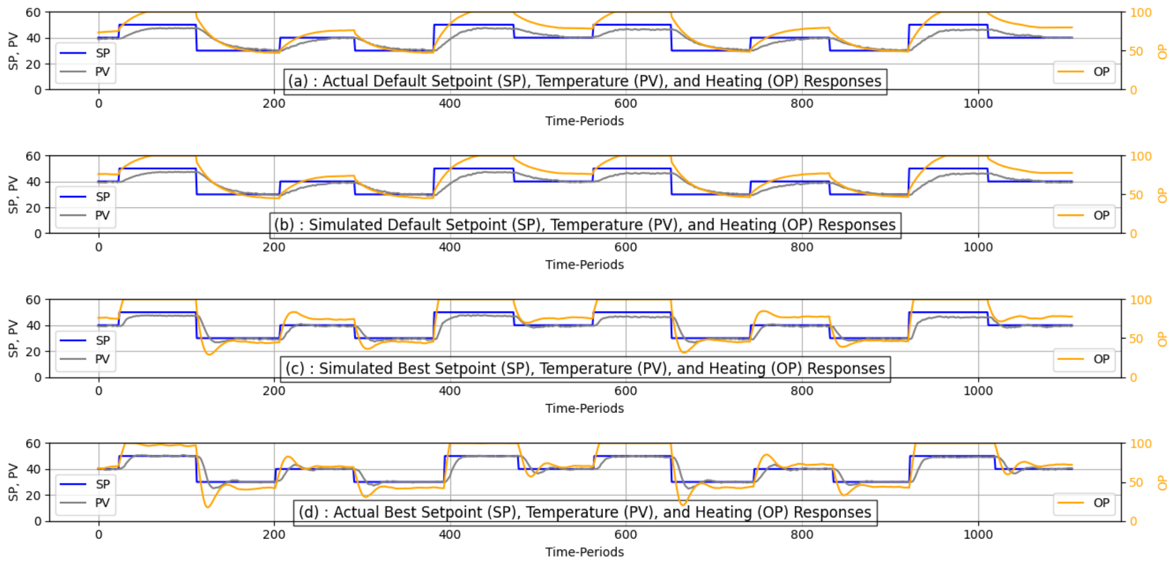

Figure 4. PCT40 Temperature PID Retuning Plots.

In figure 4a we plot the actual default PID controller's closed-loop system behavior with twelve (12) setpoint step-changes trending the setpoint (SP), process variable (PV) and controller output (OP) responses for a total of 1108 time-periods with a 10 second time-period duration as mentioned. The standardized 2-norm of the output-error (SP,t - PV,t) is 0.168450 and 0.023042 for the standardized 2-norm input-move-error (OP,t - OP,t-1). Figure 4b is the simulated version of this plot with the default PID tuning parameters and simulating after fitting an over-parameterized SISO ARX dynamic model with a five (m = 5) degree or order output lag, four (n = 4) degree input lag and a three (k = 3) degree of dead-time corresponding to a 30 seconds of time-delay which aligns with the input-output model that has the smallest or minimum sum-of-squares of errors (SSE) i.e., sum of (y,t - yp,t)^2. The simulated standardized 2-norm output-error and input-move-error are 0.166949 and 0.025011 respectively which closely matches the 2-norm errors found in figure 4a validating the SISO ARX model.

Figure 4c shows the simulation of the closed-loop system using the SISO ARX model which was identified and estimated from the data in Figure 4a. The best PID controller settings were determined by our PID retuning methodology yielding the following best PID settings of Kp = 0.3125 = 100 / (80 / 100) / (200 - (-200)) and Ti = 7.1 seconds. For the retuning, the standardized 2-norm input-move-error (IME) upper limit was set to double of the actual standardized 2-norm IME, approximately 0.05 (i.e., ~2 * 0.023042). The standardized 2-norm of the output-error (SP,t - PV,t) is 0.118537 and 0.051203 for the standardized 2-norm input-move-error (OP,t - OP,t-1) which shows a close agreement with the chosen input-move-error upper limit and has decreased the output-error by at least 29% (i.e.,(0.168450 – 0.118537) / 0. 168450 * 100) even though we have increased the input-move-error variation by over 100%. Figure 4d validates these results when the PCT40 closed-loop experiment is repeated less than two-hours after the first closed-loop experiment is performed for figure 4a. The standardized 2-norm output- and input-move-error are 0.123598 and 0.053948 respectively. These 2-norm results are quite similar to those anticipated from figure 4c.

5.5 Temperature PID Control with TCLab

The Arduino-based Temperature Control Lab (TCLab) comprises two sets of heaters, transistors, and a temperature sensor that collectively form the control process. The heaters act as the process generating heat while the transistors serve as actuators controlling the power supplied to the heaters. The temperature sensor functions as the measurement device providing real-time feedback on the temperature of the system. Together, these components enable sufficient control of the temperature within the lab environment showcasing a closed-loop control system where the sensor feedback is used to adjust the actuator output and maintain the desired temperature setpoint Park et al. (2019)[28]. The experiment began with a setpoint of

40°C, and the setpoint was nominally and automatically perturbed between 5 to 20 minutes as shown varying between 30°C and 50°C.

In figure 4a we plot the actual default PID controller's closed-loop system behavior with nine (9) setpoint step-changes trending the setpoint (SP), process variable (PV) and controller output (OP) responses for a total of 530 time-periods with a 10 second time-period duration or sampling interval and the default PID settings of Kp = 10.0 and Ti = 50 seconds with no derivative or anticipatory action (Td = 0.0). The standardized 1-norm of the output-error (SP,t - PV,t) is 1.826942 and 3.891286 for the standardized 1-norm input-move-error (OP,t - OP,t-1). Figure 4b is the simulated version of this plot with the default PID tuning parameters and simulating after fitting an over-parameterized SISO ARX dynamic model with a five (m = 5) degree or order output lag, four (n = 4) degree input lag and two (k = 2) degrees of dead-time corresponding to 20 seconds of time-delay which aligns with the input-output model that has the smallest or minimum sum-of-squares of errors (SSE) i.e., sum of (y,t – yp,t)^2. The simulated standardized 1-norm output-error and input-move-error are 1.838361 and 4.021016 respectively which sufficiently matches the 1-norm errors found in figure 4a validating the SISO ARX model.

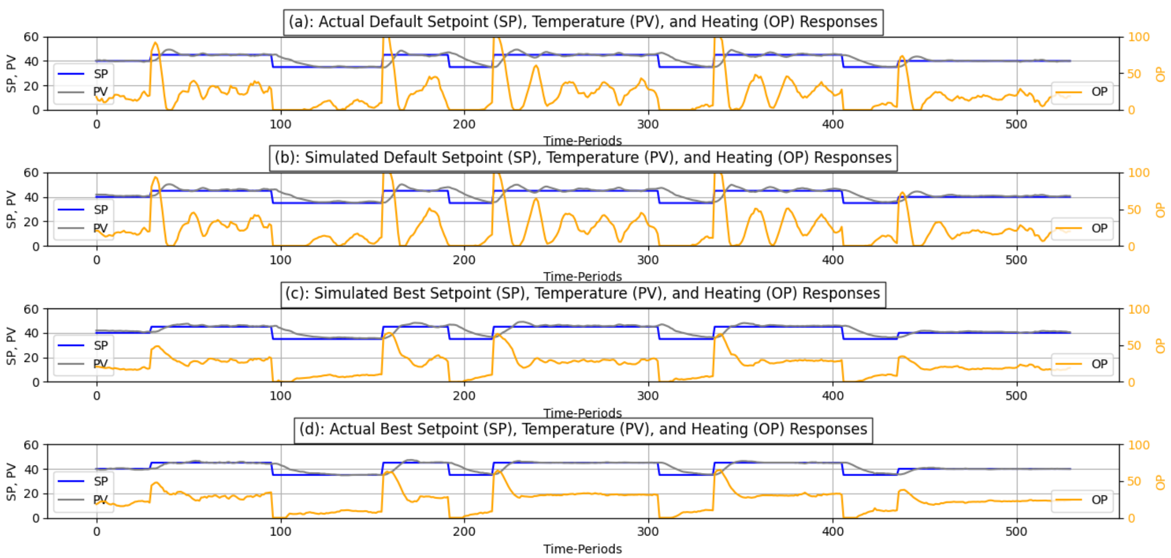

Figure 4. TCLab Temperature PID Retuning Plots.

Figure 4c shows the simulation of the closed-loop system using the SISO ARX model which was identified and estimated from the data in Figure 4a. The best PID controller settings were determined by our PID retuning methodology as Kp = 4.6 and Ti = 115 seconds. For the retuning, the standardized 1-norm input-move-error (IME) upper limit was set to circa one half of the actual standardized 1-norm IME of 1.95 (i.e., ~ 3.891286 / 2). The standardized 1-norm of the output-error (SP,t - PV,t) is 1.742377 and 1.601218 for the standardized 1-norm input-move-error (OP,t - OP,t-1) which shows relatively close agreement with the chosen input-move-error upper limit and has slightly decreased the output-error by almost 5% i.e., (1.826942 - 1.742377) / 1.826942 * 100 even though we have decreased the input-move-error variation by over 100%. Figure 4d validates these results when the TCLab closed-loop experiment is repeated less than two-hours after the first closed-loop experiment is performed for figure 4a. The standardized 1-norm output- and input-move-error are 1.521849 and 1.445109 respectively. These 2-norm results are similar to those anticipated from figure 4c except that the further reduction in the output-error variation seems to be somewhat under-predicted but obviously in the same direction of improvement as expected by the best simulated PID results.

5.6 Level PI Control of Surge Drum with Closed-Loop Dynamic Simulation

In this closed-loop dynamic simulation, a simple horizontal surge drum process system with water as its fluid is designed to regulate its liquid level or holdup h_t under feedback control with a pump-drained outlet flow as shown in Figure 5. The flowsheet configuration is taken from Kelly (1998)[29] and the first principles modeling and parameters are provided by Jang (2016)[30] but altered with a pump-drained outlet flow with a separate flow control loop instead of being gravity-drained and thus requiring no details on the level control valve. The inlet flow q^i_t enters at the top and for this dynamic simulation is used as a random walk load disturbance i.e., integrating white noise as discussed in Kelly (1998)[29]. The outlet flow's $q^o{}_t$ is under perfect flow control (FC) with its setpoint cascaded from the PI level controller's (LC) output as shown. Proper tuning of the LC involves setting the appropriate proportional gain (Kp) and integral or reset time (Ti) settings to achieve what is known as "level flow smoothing" or "averaging level control". The closed-loop feedback dynamic simulation is programmed in IMPL-DATA© from Industrial Algorithms Limited as an Intel Fortran user-coded external data function combining the process system equations discretized and integrated via the implicit or forward Euler's method with the classic PI equations in velocity-form where the output-error is used in both PI terms known as "Equation A". This dynamically simulated system is referred to as the actual system or apparatus which in reality is a digital twin or cyber-physical representation in lieu of a true physical system.

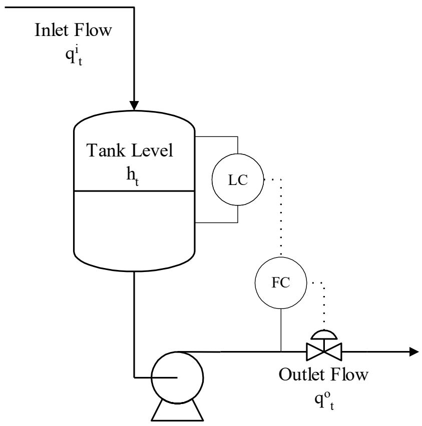

Figure 5. Dynamically Simulated PI Level Controlled Surge Drum Apparatus

As previously discussed, our methodology involves estimating the process dynamics via SISO ARX, however for integrating processes such as this we follow the technique from Kelly (1998)[29] which single differences the process input ($\nabla q^o t$) and double differences the process output ($\nabla^2 h_t$). By itself, we only need to estimate the process gain and dead-time as the process dynamics are already known due to the integrating nature of the process. The process gain is then theoretically equal to the sampling-interval or scan- or time-period duration divided by the cross-sectional area of the horizontal vessel and negative as the process input is the outlet flow i.e., an increase in the flow out decreases the drum holdup where the process flows are in flowrate unit-of measure e.g., m^3/s. The process delay is identified by straightforwardly estimating multiple SISO ARX model fits and selecting the one with the least sum-of-squares of residuals or errors (SSE) or largest coefficient of determination (R^2) for example also discussed in Kelly (1998)[29].

For this example, the horizontal surge drum cross-sectional area is 0.1 m^2, the level has a steady-state of 1.0 m and a minimum and maximum of 0.0 and 2.0 m. The steady-state outlet flow and inlet flow is 0.001 m^3/s and its minimum and maximum are 0.0 and 0.002 m^3/s. The default PI level controller has a proportional gain of -1.5 and an integral time of 1,200 seconds where the PI level controller was run, cycled or executed every 10.0 seconds and the total time-profile for the closed-loop dynamic simulation is 5,000 sampling-instants or 50,000 seconds. The inlet flow random walk load disturbance's white noise sequence generator uses an arbitrary random seed and a standard-deviation of 10% of the inlet flow's steady-state i.e., 0.1 * 0.001 = 0.0001 m^3/s as specified by Jang (2016)[30]. The SISO ARX with the least SSE identified and estimated has a dead-time of 10.0 seconds corresponding to a single scan-interval or simulation time-period and the process gain estimate is -0.209 based on the level controller's fractional and dimensionless output response which varies from 0.0 to 1.0. Converting this to physical unit-of-measures, the actual process gain is -0.209 / (0.002 – 0.000) = -104.5 s/m^2 which closely approximates the theoretical value of -10.0 / 0.1 m^2 = -100.0 s/m^2.

The first sub-plot Figure 6a displays the simulated actual closed-loop response using the dynamic simulation apparatus with the default PI settings and no setpoint changes, perturbations, dithering nor disturbances. The second sub-plot Figure 6b shows the simulated version using the SISO ARX model, the same default PI level controller and replaying back the same load disturbance derived from the simulated actual minus the predicted level or holdup response i.e., the regression residuals form the SISO ARX fit. As can be observed the two sub-plots are in close agreement indicating that the SISO ARX model seems representative. The third sub-plot Figure 6c presents the best PI level retuning results where the retuning has -0.2 for the controller gain (Kp) and 10,000 (Ti) seconds which is essentially a proportional-only controller as seen by the large swings or deviations in the process output response absorbed by the excess holdup capacity in the surge drum as expected which ultimately reduces the variation in the outlet flow consistent with the notion or concept of level flow smoothing or averaging level control. The simulated best sub-plot is the last sub-plot for Figure 6d and is virtually the same as the third as anticipated. The reason for the very close agreement is due to the fact that the pump-driven outlet flow translates into a linear ordinary differential equation (ODE) for the process whereas a gravity-driven outlet flow would yield a nonlinear ODE given that the outlet flow would be a function of the square-root of the height or level. Since the SISO ARX model is only a linear dynamic approximation to the real process system, the process model identification and estimation is thus more accurate.

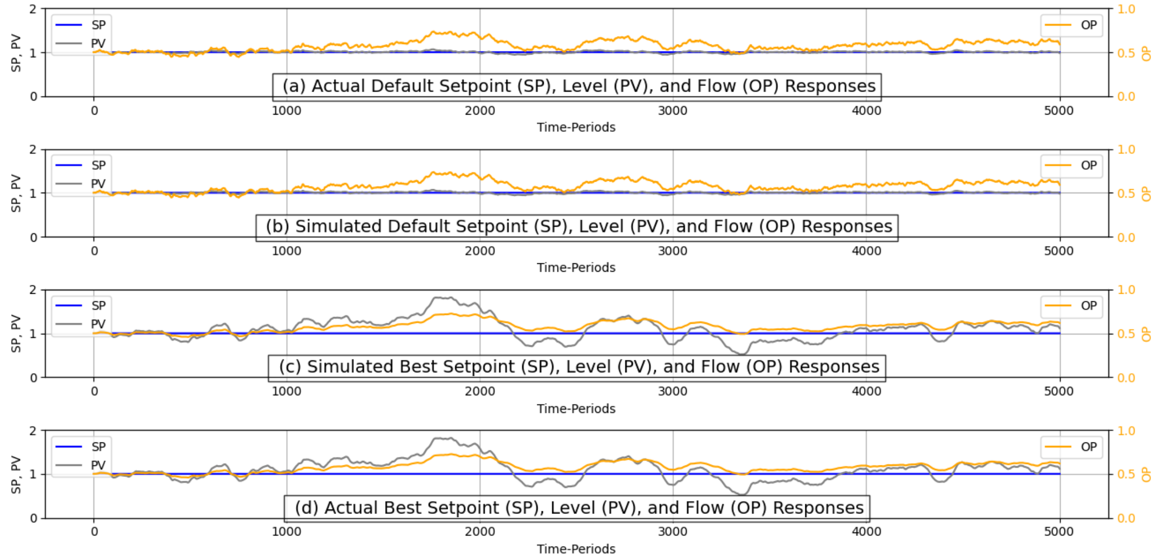

Figure 6. Level PI Controller Retuning Plots with Level Upper Limit of 2.0 m.

In Figure 7 we restrict the large variations in the holdup response that vary from 0.0 to 1.5 m instead of 2.0 m inside our PID retuning technique which simply excludes all internally generated PID simulation scenario or situation results which violate at any time-instant within the retuning feedback simulation time-horizon or -profile level values that are greater than 1.5 m and less than 0.0 m. The new best tuning PI settings are now -0.2 and 1,850 seconds where it is clearly demonstrated that the previous deviation of 1.8 m found in the fourth sub-plot of Figure 6 is now attenuated to less than or equal to 1.5 m by reducing the reset or integral time allowing for quicker adjustment of the outlet flow.

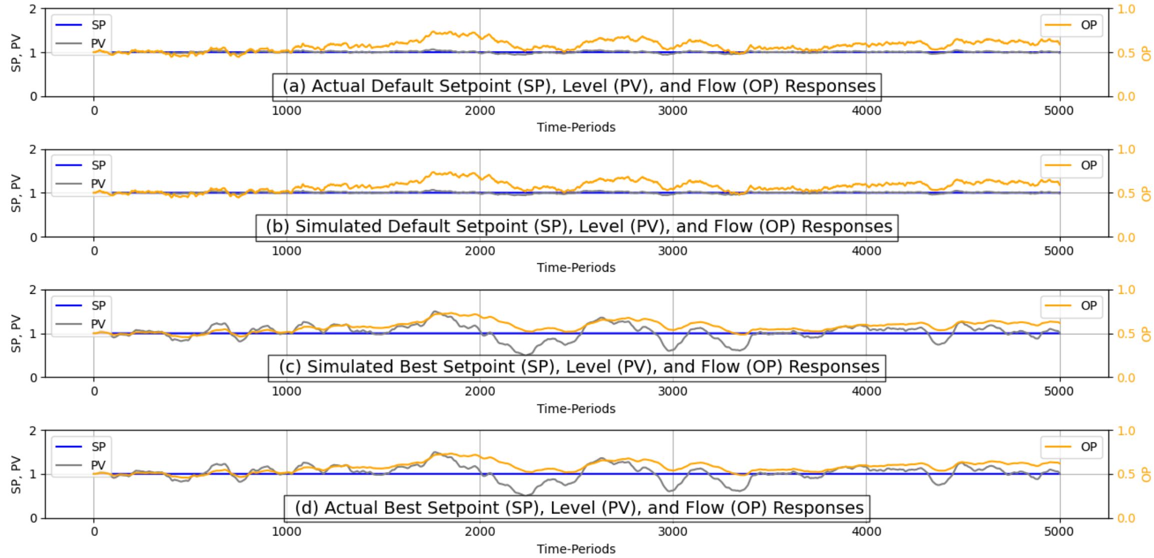

Figure 7. Level PI Controller Retuning Plots with Level Upper Limit of 1.5 m.

5. Discussions

The experimental results presented in this study demonstrate the effectiveness of the proposed sample-based retuning method for PID controllers. The key findings and their implications are discussed below.

5.1 Performance Evaluation

The method showed reliable performance across different pilot plants, specifically in flow, pressure, and temperature control loops. The retuning process effectively minimized the integrated squared error (ISE) and mitigated the rate of change, which are crucial for maintaining stability in industrial processes. The exhaustive search method applied in this study provided a straightforward approach that can be easily implemented in real-world operations using digital twin simulations and optimizations.

5.2 Comparison with Existing Methods

Unlike traditional gradient-based optimization techniques, which can often stall at local minima due to the non-convex nature of the problem, our brute-force search method demonstrated robust global minimum searching capabilities. This is particularly important for processes with complex dynamics where local optimization methods may fail to give optimal tuning parameters. The higher accuracy achieved in PID tuning using the exact same sequences of setpoint changes and load disturbances during process model evaluation highlights the superiority of our method over conventional approaches.

5.3 Practical Implications

Implementing the proposed retuning method in industrial settings can lead to significant improvements in process control. By utilizing closed-loop feedback data and digital twin simulations, operators can retune PID parameters more accurately and efficiently. This method also allows for the replay of past setpoint and load disturbances, considering residuals of estimation as unmeasured load disturbances, thus providing a more comprehensive evaluation of the controller performance under various operational conditions.

5.4 Limitations and Future Work

While the results are promising, the method's reliance on extensive computational resources for brute-force search may pose a challenge in environments with limited computational capabilities. Future research should focus on optimizing the search algorithm to reduce computational overhead without compromising accuracy. Additionally, further validation of the method across a wider range of industrial processes and conditions will be essential to generalize its applicability. In summary, the sample-based retuning method for PID controllers presented in this study offers a reliable and practical approach for improving process control performance. By addressing the limitations of existing methods and leveraging digital twin simulations, this approach holds significant potential for enhancing industrial process stability and efficiency.

6. Conclusion

This research presents a robust data-driven methodology for PID controller retuning using a brute-force sample-based search applied to surrogate models identified from routine closed-loop operating data. By leveraging ARX models within a digital twin framework, the proposed approach enables systematic retuning without disrupting operations or requiring open-loop testing, which is an advantage for modern industrial environment focused on safety, uptime and cost-efficiency.

The method was validated across a diverse set of processes, including flow, pressure, and temperature control loops, as we as a simulated integrating level control system. In all cases, the methodology achieved substantial reductions in output-error norms while maintain or improving actuator efficiency, confirming its practical viability for both self-regulating and integrating process dynamics. Importantly, the simulation-based approach enables tuning decisions to be informed by realistic, process-specific disturbances and setpoint changes, enhancing confidence in real-world performance gains.

Compared to traditional gradient-based optimization and heuristic methods, the brute-force search technique avoids local minima and reliably finds globally optimal solution within user-defined bounds. While the computational cost of an exhaustive search remains a consideration, this is mitigated by its offline nature and potential for parallel execution. The method is especially valuable in supervisory control architectures where controllers must be tuned or retuned periodically in response to process drift, equipment aging, or changing production targets.

Future work should explore hybrid optimization schemes that combine the robustness of brute-force search with the speed of local gradient-based methods to reduce computational load. Moreover, expanding the method's application to Multivariable (MIMO) system and integrating robustness margins into the tuning objectives would further enhance its industrial relevance. The integration of online monitoring to automatically trigger retuning based on model mismatch or performance degradation offers a promising pathway towards fully autonomous control loop optimization. In summary, this methodology offers a scalable and reliable path to smarter PID control—bridging the gap between conventional tuning rules and modern, data0driven control strategies.

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